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Citation: Physics Today **69**(9), 70 (2016); doi: 10.1063/PT.3.3307
View online: http://dx.doi.org/10.1063/PT.3.3307
View Table of Contents: http://scitation.aip.org/content/aip/magazine/physicstoday/69/9?ver=pdfcov
Published by the AIP Publishing

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In many biological systems, structures all grow at the same rate. That phenomenon of proportionate growth has now been observed in a nonequilibrium physical system.

The natural world is full of patterns that spontaneously emerge from featureless environments. From microscopic snowflakes to large-scale river networks, pattern formation leads to systems of extraordinary intricacy and beauty.

Instabilities are central to pattern formation; complex patterns emerge when a system is driven to an unstable state in which small perturbations evolve to create large-scale structures. However, the details of the evolution following the instability onset can vary widely. They depend on the system's intrinsic symmetries or the growth conditions set by the environment; the result is a variety of patterns, diverse in both their organization and their length scales. Understanding how a system spontaneously selects its overall structure as it is driven out of equilibrium remains a major scientific challenge.

Crucial to many advancements in the field was the discovery of the viscous fingering instability, which occurs when a fluid displaces another of higher viscosity either in a porous medium or in a narrow gap. A convenient model environment for precision studies of interfacial pattern formation is the Hele-Shaw cell—two parallel plates separated by a thin gap of thickness b, in which the fluids are injected through a hole in the center of one of the plates. In that system, the encroaching low-viscosity fluid forms fingers that grow and then split if they become too wide. (See also PHYSICS TODAY, October 2012, page 15.)

The viscous fingering instability has received enormous attention since 1958 when Philip Saffman and G. I. Taylor showed that both the onset of the instability and the subsequent branching of growing fingers are governed by the size scale λ_c of the fastest-growing perturbation. That most unstable scale is determined by a competition between the interfacial tension σ, which stabilizes small perturbations, and stresses that drive the instability. Those stresses depend on the interfacial velocity V and the viscosity difference $\Delta\eta = \eta_{out} - \eta_{in}$ between the displaced outer fluid and the invading inner fluid. Thus, $\lambda_c = \pi b\sqrt{\sigma/\Delta\eta V}$. The scale λ_c sets the characteristic width of the first generation of fingers and the condition for tip splitting, which occurs once a finger has grown to a width of $2\lambda_c$.

An independent length scale

In collaboration with Radha Ramachandran, we have discovered that the above single-scale description does not fully characterize the patterns that emerge in the Hele-Shaw cell. It fails to account for some of the global features that evolve after the fingers have developed. As illustrated by panel a of the figure, systems with identical λ_c can evince a rich variety of shapes with distinct global characteristics. The images in the top row of the panel show patterns formed when a water–glycerol mixture displaces a more viscous oil at a constant λ_c of 2.8 ± 0.2 mm.

The unambiguously distinct patterns are created by varying the viscosity ratio η_{in}/η_{out}. They all include an interior region in which the outer fluid is completely displaced. As the viscosity ratio increases, the inner circular region grows and the finger length shrinks; the less viscous fluid more efficiently displaces its more viscous partner. Remarkably, since the finger length is controlled by η_{in}/η_{out} and the width by λ_c, the fingers' two dimensions are decoupled from each other.

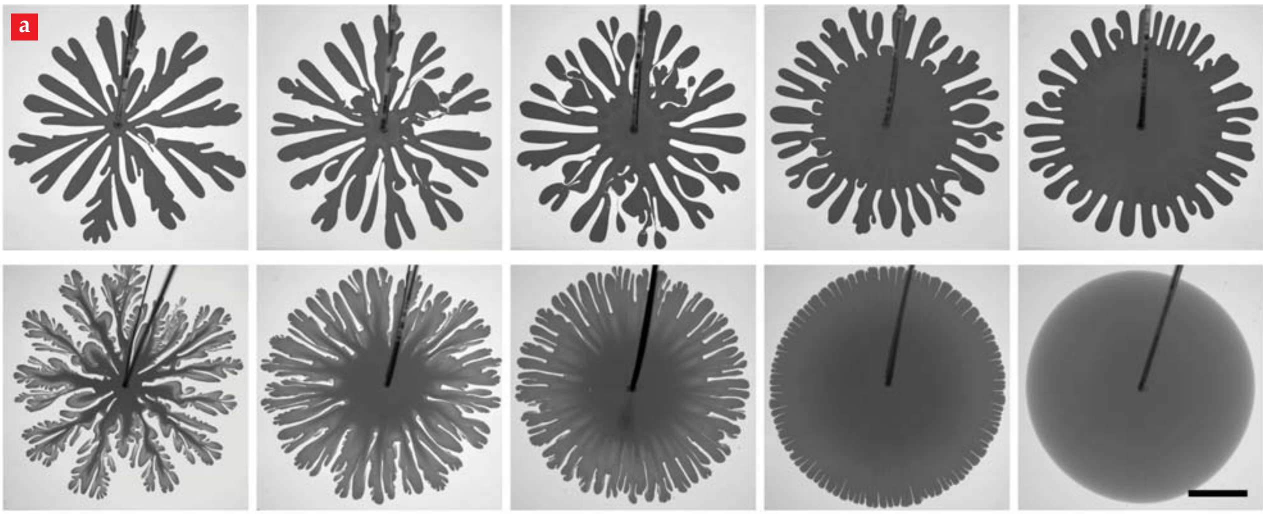
Removing surface tension stabilizes interface

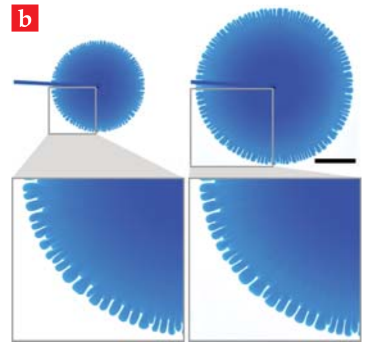
The decrease in the relative finger length with increasing η_{in}/η_{out} is a general feature of the fingering instability. As the bottom row in panel a of the figure shows, the decrease is also observed when the two fluids are miscible—in other words, when the interfacial tension σ approaches zero. In that case, the most unstable scale is minimal, but it is not strictly zero as the explicit formula above would indicate; instead, in the small-σ limit, λ_c is set by b, the gap thickness between the two plates.

The vanishing of σ corresponds to the elimination of the only stabilizing term in the system. Counterintuitively, the elimination of interfacial tension stabilizes the fingering pattern. This is illustrated in the comparison between panel a's upper and lower rows, which show, respectively, pairs of patterns formed at significant σ and nearly zero σ for five viscosity ratios. For each pair, the stable central region is larger and the finger length smaller in miscible fluids with tiny σ. Indeed, for the largest η_{in}/η_{out}, the system is completely stable—no fingering is observed.

It turns out that the key to the unexpected stability lies in the three-dimensional flows that exist in the miscible fluids. Instead of fully displacing the outer fluid, the inner fluid penetrates as a tongue into the outer fluid. The lower row of panel a shows evidence for the formation of such tongues—the varying shades of gray in the fingering patterns indicate more or less incomplete displacements of the outer fluid.

The subtle balance of stable and unstable growth in miscible fluids leads to a second unexpected phenomenon. When the viscosity ratio is close to the boundary of the stable regime, the instability is suppressed after the first generation of fingers has





STRIKING FORMS CAN EMERGE when a low-viscosity fluid displaces one of higher viscosity. **(a)** The patterns here were created by two immiscible fluids (top row) and two miscible fluids (bottom row) for different viscosity ratios: from left to right, 0.0026, 0.025, 0.073, 0.24, and 0.37. As the ratio increases, the fingers become progressively shorter and an inner circular region of complete displacement becomes ever bigger. Over the entire range of ratios, the fingers are shorter for the miscible fluids; evidently, the elimination of interfacial tension stabilizes the pattern. **(b)** When miscible fluids are used and their viscosity ratio is high enough, a novel proportionate growth pattern results. The top row shows two snapshots of the growth for a system in which the viscosity ratio is 0.185. The sections in the squares are zoomed to give the bottom images, which have the same outer radius. Those zoomed images are essentially indistinguishable; evidently, the fingers grow in direct proportion to the overall pattern radius. The scale bars correspond to 4 cm.

developed. That is, no further splitting is observed even as the finger width becomes much larger than $2\lambda_c$, the value at which one would expect a finger to branch. A fundamentally different type of growth is observed, one that is distinct from the commonly found growth morphologies.

Proportionate growth

The patterns we have observed at the edge of stability in miscible fluids grow without tip splitting or side-branch formation, another common instability mode. Instead, the structures that form grow at nearly the same rate in all directions; thus the overall growth does not change shape (see panel b of the figure). That type of growth, in which a pattern is composed of distinguishable structures all growing at the same rate, is called proportionate growth. It had not previously been empirically observed in a physical system, though it had been seen in a cellular automaton devised by Tridib Sadhu and Deepak Dhar (see the additional resources). Other than that, proportionate growth has been observed only in the biological world, where it is common. A spectacular example is the growth of mammals; as a baby mammal grows, different body parts grow at nearly the same rate and thus in direct proportion to each other. Biologists have long pondered how the body organizes that synchronized growth.

In the viscous fingering instability, proportionate growth occurs because the system becomes unstable only once, to produce distinguishable structures, but then turns off the instability mechanism so that no further generations can develop. That mechanism is different from what happens in the Sadhu–Dhar cellular automaton, which follows unchanging preset rules. Now that we have found a physical system with proportionate growth, we and other physicists can access that type of growth experimentally under controlled conditions.

Nature rarely closes the book on any complex phenomenon, and careful investigation of new regimes is often repaid with surprising results. The work presented here shows that even for a well-studied problem such as viscous fingering, experiments can reveal novel and counterintuitive behavior. Future experiments and simulations coupled with new theoretical understandings will clarify any relevance of viscous fingering to proportionate growth in biology and the importance of new length scales for controlling technological applications in which fluid displacements are essential.

We are grateful to NSF for support of this research.

Additional resources

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